Ma 116c Homework #2

Due Thursday, May 9th at 1:00pm

1) Show that the following assertions are absolute (for any transitive model of a sufficient fragment of ZFC) by writing each one as a Δ_0-formula.

   i.) $z = \langle x, y \rangle$,

   ii.) z is an ordered pair,

   iii.) R is a relation,

   iv.) $C = A \times B$.

For exercises (2) - (4), M denotes a countable transitive model of ZFC and $\mathbb{P}$ denotes a partial order. If $\mathbb{P} \in M$, then G denotes a filter that is generic for $\mathbb{P}$ over M.

2) $p \in \mathbb{P}$ is called an *atom* if

$$\neg \exists q, r \in \mathbb{P}(q \leq p \wedge r \leq p \wedge q \perp r)$$

   $\mathbb{P}$ is *non-atomic* if $\mathbb{P}$ has no atoms. Show that if $\mathbb{P} \in M$ and p is an atom of $\mathbb{P}$, then there is a filter $G \in M$ such that $p \in G$ and G intersects *all* dense subsets of $\mathbb{P}$.

3) Suppose that $\mathbb{P} \in M$ is non-atomic. Show that $\{G : G \text{ is } \mathbb{P}\text{-generic over } M\}$ has cardinality 2^ω.

4) If $\sigma, \tau \in M^{\mathbb{P}}$, show that $\sigma_G \cup \tau_G = (\sigma \cup \tau)_G$.