Ma 116c Homework #3

Due Thursday, May 23rd at 1:00pm

Let M be a fixed ctm of ZFC.

1) As in class, let $\kappa \in M$ be such that (κ is an uncountable cardinal)^M, and let $\mathbb{P} = \mathrm{Fn}(\kappa \times \omega, 2)$. Let G be $\mathbb{P}$-generic over M, and let $f_G = \bigcup G$ be the associated function derived from G from $\kappa \times \omega$ to 2. Also as in class, for $\alpha \in \kappa$ define $f_\alpha : \omega \to 2$ to be the α-th section of f_G defined by $f_\alpha(n) = f_G(\alpha, n)$.

   Show that *all* of the functions f_α are new in the extension, i.e. for every $\alpha \in \kappa$ we have $f_\alpha \in M[G] \setminus M$.

2) Suppose $\mathbb{P} \in M$ and G is a filter for $\mathbb{P}$. Show that the following are equivalent:

   i.) $G \cap D \neq \emptyset$ whenever $D \in M$ and D is dense in $\mathbb{P}$,

   ii.) $G \cap A \neq \emptyset$ whenever $A \in M$ and A is a maximal antichain in $\mathbb{P}$.

3) Assume $f : A \to M$ and $f \in M[G]$. Show that there is $B \in M$ such that $f : A \to B$.

   Hint: Find τ such that $f = \tau_G$ and let

$$B = \{b : \exists p \in \mathbb{P}(p \Vdash \check{b} \in \mathrm{ran}(\tau))\}.$$