Ma/CS 117a HW #2

Due Tuesday, October 22nd, at 1pm

1) Let $A(n, x)$ be the Ackermann function. Consider the graph $G \subseteq \mathbb{N}^3$ of A, i.e. the ternary relation

$$G(n, x, z) \Leftrightarrow A(n, x) = z$$

Show that G is primitive recursive.

2) Consider the following "stack algorithm" for computing the Ackermann function $A(n, x)$:

Input: A pair $(n, x) \in \mathbb{N}^2$.

Algorithm: Given a sequence $s = (n_1, \cdots, n_k)$ of natural numbers, do the following:

–If $k = 1$, i.e., $s = (n_1)$, *Stop*: n_1 is the *output*.

–If $k \geq 2$, write $s = t\,\hat{}\,(n', x')$ (where here $\hat{}$ denotes concatenation), with t a finite sequence of numbers, perhaps empty. Then do the following:

- If $n' = 0$, replace s by $s' = t\,\hat{}\,(x' + 1)$.
- If $n' > 0, x' = 0$, replace s by $s' = t\,\hat{}\,(n' - 1, 1)$.
- If $n' > 0, x' > 0$, replace s by $s' = t\,\hat{}\,(n' - 1, n', x' - 1)$.

Show that for each input (n, x) this algorithm terminates after finitely many steps (i.e., we reach a sequence of length 1) and the output is the value $A(n, x)$.

3) Show that the following partial function $f : \mathbb{N} \to \mathbb{N}$ is recursive: $f(n) = 1$, if there is a sequence of n consecutive 7's in the decimal expansion of the square root of 2; $f(n)$ is undefined otherwise.