

Ma/CS 117a HW #4

Due Tuesday, November 5th, at 1pm

- 1) We have shown in class that for each (partial) recursive function $f : \mathbb{N}^k \rightarrow \mathbb{N}$ there is a TM M on $\{1, \sqcup, \blacksquare\}$ such that $f = \varphi_M^k|_{\mathbb{N}^k}$. Let T_n be the class of all TM on $\{1, \sqcup, \blacksquare\}$ having at most n states. Clearly T_n is finite. For each TM M on $\{1, \sqcup, \blacksquare\}$, let P_M be defined by

$$P_M = \begin{cases} \text{the number of 1's appearing in the} \\ \text{output, when the input in } M \text{ is } \emptyset, \\ \text{and } M \text{ halts on that input;} \\ 0, \text{ otherwise.} \end{cases}$$

So $P_M \in \mathbb{N}$. Let

$$B(n) = \max\{P_M : M \in T_n\}.$$

Thus $B : \mathbb{N} \rightarrow \mathbb{N}$. We call B the *busy beaver* function. For each n , $B(n)$ is the maximum number of 1's that a TM with at most n states can print before halting, when started at the empty tape.

Prove that B is *not* recursive.

(*Hint.* Assume, towards a contradiction, that the TM M_B computes $f(n) = B(n) + 1$. Let the TM M_0 take the input

$$\cdots * \underbrace{1 \dots 1}_x, \underbrace{1 \dots 1}_y, * * \dots$$

into the output

$$* * \underbrace{1 \dots 1}_{xy} * * .$$

Let M_k be a TM which on the empty input

$$\cdots * * * \dots$$

produces output

$$\cdots * * \underbrace{11 \dots 1}_k, \underbrace{11 \dots 1}_k, * * * \dots$$

and consider the TM M' which is the “concatenation” of M_k followed by M_0 followed by M_B .)

- 2) Show that there is no Markov algorithm $\mathcal{A}$ on the alphabet $A = \{1\}$ (note that A does not contain “,”) such that for each $n \in \mathbb{N}$, $\mathcal{A}(n) = 2n$. Here n stands for $11 \dots 1$ (n times). (We know of course that there is a MA $\mathcal{A}'$ on some alphabet $B \supseteq \{1\}$ such that for each $n \in \mathbb{N}$, $\mathcal{A}'(n) = 2n$.)