Ma/CS 117a HW #5

Due Tuesday, November 12th, at 1pm

1) Find explicitly a Markov algorithm $\mathcal{A}$ on an alphabet B containing the symbol 1 and the comma symbol , which computes the function $f(m,n) = m - n$, if $n < m$; $= 0$, if $n > m$. That is, if the input to $\mathcal{A}$ is $\underbrace{11\ldots1}_{m},\underbrace{11\ldots1}_{n}$, the output is $\underbrace{11\ldots1}_{f(m,n)}$.

2) Show that there is a partial recursive function $f : \mathbb{N} \to \mathbb{N}$ for which there is no *total* recursive extension $g \supseteq f$, i.e., there is no total recursive $g : \mathbb{N} \to \mathbb{N}$ such that for any n in the domain of f we have $f(n) = g(n)$.

   Hint. Use diagonalization on a universal recursive function.

3) Show that $R \subseteq \mathbb{N}$ is r.e. iff R is finite or there is a 1-1 total recursive function $f : \mathbb{N} \to \mathbb{N}$ with range$(f) = R$. (The difference here from the definition is the hypothesis that f is 1-1.)

4) Show that $R \subseteq \mathbb{N}$ is recursive iff R is finite or there is a strictly increasing recursive total function $f : \mathbb{N} \to \mathbb{N}$ with range$(f) = R$.