Ma/CS 117a HW #6

Due Tuesday, November 19th, at 1pm

1) Show that every infinite r.e. set $R \subseteq \mathbb{N}$ contains an infinite recursive subset $P \subseteq R$.

   Hint. On the last set you showed every r.e. set is the range of a 1-1 total recursive function.

2) (The *Selection Theorem* for r.e. sets.) Show that if $R \subseteq \mathbb{N}^2$ is r.e., then there is a partial recursive function $f : \mathbb{N} \to \mathbb{N}$ such that

   i) $f(x) \downarrow \Leftrightarrow \exists y R(x, y)$,

   ii) $\exists y R(x, y) \Rightarrow R(x, f(x))$.

3) (The *Reduction Theorem* for r.e. sets.) Show that if $A, B \subseteq \mathbb{N}^n$ are r.e., then there are r.e. sets $A^*, B^* \subseteq \mathbb{N}^n$ such that

$$A^* \subseteq A, B^* \subseteq B, A^* \cap B^* = \emptyset, A^* \cup B^* = A \cup B.$$

4) (The *Separation Theorem* for co-r.e. sets.) A set is *co-r.e.* if its complement is r.e. Show that if $A, B \subseteq \mathbb{N}$ are co-r.e. sets and $A \cap B = \emptyset$, then there is a recursive set $C \subseteq \mathbb{N}$ such that

$$A \subseteq C, B \cap C = \emptyset.$$

   Hint. Use 3).

5) Consider the set $H \subseteq \mathbb{N}$ consisting of all numbers e that code a Turing machine on the alphabet with symbols 1 and , that terminates on the input $\cdots * **, * * * \ldots$ (which represents the numeric input 0). Show that H is r.e. but not recursive.