Course overview:

- class is an intro to writing proofs.
- we'll take a tour of subjects:

Basic set theory, logic, number theory, combinatorics, etc.

- So: what is "doing math"?
  - ↳ not just calculating!
- Roughly: it's about investigating mathematical objects (e.g. integers, right triangles, continuous functions) by proving the truth or falsity of mathematical statements about these objects (e.g. "every continuous function is differentiable").
- mathematical objects are described by precise definitions, e.g.:

Def'n: a prime number is a positive integer p, greater than 1, such that if n is a positive integer that divides p then $n = 1$ or $n = p$.

Some non-def'ns: "A line is a flowing point"

"A point is a place without extension"

- Emerson

↳ suggestive, poetical ... but not precise.

- mathematical statements (or propositions) are declarative sentences (concerning math'l objects) that are either true or false (i.e. they have a truth value)

e.g:

Prop'n 1: (Euclid) There are infinitely many prime numbers (or, in Euclid's words: "There are more primes than found in any list of primes")

↳ prop'n 1 is either true or false: either there are infinitely many primes, or not (in fact: there are)

↳ establishing the truth of a proposition requires a proof.

- roughly: a proof is a sequence of logical deductions from axioms or previously proved statements whose conclusion is the prop'n in question.

- many methods of proof. One is to argue by contradiction.

Proof of prop'n 1: - Sps ("suppose") toward a contradiction that prop'n 1 is false i.e. that there are finitely many primes

- then we can list them as:

$p_1, p_2, \dots, p_n$

- consider the integer

$$N = p_1 \cdot p_2 \cdots p_n + 1$$

obtained by multiplying all the primes in our list and adding 1.

e.g. maybe 2, 3, 5, 7 are the only primes & in this case N would be $2 \cdot 3 \cdot 5 \cdot 7 + 1 = 211$

- Observe: if we divide N by any of the primes $p_1, \dots, p_n$ we leave a remainder of 1.

- Hence: N is not divisible by any of the primes $p_1, p_2, \ldots, p_n$
- So: either N itself is prime, or there is another prime p not among $p_1, \ldots, p_n$ that divides N

(I'm using here that every integer is either prime or divisible by some prime)

- either way: there must be another prime not among $p_1, p_2, \ldots, p_n$

↳ a contradiction as we assumed these were all of the primes.

- Hence our assumption was false
- Hence there are infinitely many primes ✓

$\boxed{\text{Sets}}$ – a set is a collection of objects (sometimes described by a common property)

Contor: "By a 'set' we are to understand any collection into a whole M of definite and separate objects m of our intuition and thought."

- this is an informal def'n (and, in fact, a contradictory one)
- formal def'n of set beyond scope of course.
- our approach: we'll write down several fundamental sets that we'll take for granted, then give formal def'ns of operations that allow us to build new sets from old ones.

- sets are enclosed by curly brackets

  $\{\ldots\}$

- objects in a set are called elements
- $\in$ means "is an element of"

  $\notin$ means "is not an el't of"

Ex's: 1 Let E denote the set of positive even integers.

- we also write:

  $$E = \{2, 4, 6, \ldots\}$$

then: $12 \in E$, while $1 \notin E$ and $-2 \notin E$