- e.g. $(\forall x \in \mathbb{R})(x^2 \geq 0)$ is meaningful
  $(\forall x)(x^2 \geq 0)$ is not.

↳ but then: what if we want to quantify over variables referring to sets?

- e.g. to write "For every set S, we have $\emptyset \subseteq S$."

Symbolically we might try:

$$\underbrace{(\forall S \in (\cdots))}_{\text{set of all sets??}}(\emptyset \subseteq S)$$

- but the collection of all sets is not a set (Russell's paradox)

- convention: when quantifying over set variables, we'll write statements verbally, i.e.

"For all sets S, ..."

"There exists ~~the~~ a set A, ..."

Connectives and Truth Tables

- connectives are symbols used to combine multiple statements into one.
- all our connectives will be binary (connect two statements into one) except negation which is unary.
- Truth tables tell us how the truth of a connected statement depends on the truth of its constituent statements.

Conjunction ("and")

- The conjunction of two statements P, Q is written $P \wedge Q$. ("P and Q")
- $P \wedge Q$ is true iff both P and Q are true.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex's: Let P denote "$(\forall x \in \mathbb{Z})(x+1 > x)$"

Let Q denote "97 is prime"

Let R denote "$2+2=5$"

Then P, Q are (T) but R is (F)

- hence $P \wedge Q$ is (T) while $P \wedge R$ and $Q \wedge R$ are both (F)
- written out, $P \wedge Q$ is:

$$(\forall x \in \mathbb{Z})(x+1 > x) \wedge (97 \text{ is prime})$$

inserting parentheses can clarify an expression.

$\boxed{\text{Disjunction}}$ ("or")

- The <u>disjunction</u> of two statements P, Q is written $P \vee Q$ ("P or Q")
- $P \vee Q$ is true iff <u>at least one of</u> P, Q is true.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

e.g.

$$\overset{T}{(\forall x \in \mathbb{R})(x^2 \geq 0)} \vee \overset{F}{(96 \text{ is prime})}$$

is (T) while

$$\underset{F}{(\forall x \in \mathbb{R})(x \geq 0)} \vee \underset{F}{(96 \text{ is prime})}$$

is (F).

$\boxed{\text{Negation}}$ ("not")

- our only unary connective
- <u>negation</u> of a statement P written $\neg P$ ("not P")
- $\neg P$ is true iff P is false

P	$\neg P$
T	F
F	T

<u>Ex's</u> ① $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(y^2 = x)$ is (F), hence

② $\neg(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(y^2 = x)$ is (T), hence:

③ $\neg\neg(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(y^2 = x)$ is (F) again.

④ For <u>any</u> statement P, the statement

$P \vee \neg P$ is (T)

whereas $P \wedge \neg P$ is (F)

e.g. (96 is prime) $\vee$ $\neg$(96 is prime)

is (T) while

(96 is prime) $\wedge$ $\neg$(96 is prime)

is (F).

Can use connectives in var prop'ns too! e.g. let $P(x,y)$ denote

"$(x>0) \wedge (y$ is prime$)$"

then: $P(3,5)$ is (T)

but $P(3,6)$ is (F)

also: $(\exists x,y \in \mathbb{N})\, P(x,y)$ is (T)

but $(\forall x,y \in \mathbb{N})\, P(x,y)$ is (F).

- also in def'ns, set-builder notation etc.

- e.g. if A, B are subsets of a universal set U, can define: