(both are (T): 0 has no multiplicative inverse) → "there is a real w/o a multiplicative inverse"

Negating Connected Statements

Prop'n For any statements P, Q the following logical equivalencies hold (i.e. these statements are always true)

1. $\neg\neg P \Leftrightarrow P$
2. $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
3. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$

← De Morgan's Laws

Pf: to prove, we use truth tables

①

P	$\neg P$	$\neg\neg P$	$\neg\neg P \Leftrightarrow P$
T	F	T	T
F	T	F	T

always true

$\neg\neg P \Leftrightarrow P$

$\neg\neg P$ $\quad$ P

$\neg P$

P

2 $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

- $\neg(P \wedge Q)$
  - $P \wedge Q$
    - P
    - Q
- $\neg P \vee \neg Q$
  - $\neg P$
    - P
  - $\neg Q$
    - Q

Proof of 3 is similar

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

always true

Ex's ① $\neg\neg(1+1=2)$

equiv to $1+1=2$ (both (T))

② $\neg((1+1=2) \wedge (1+1=3))$

equiv to $(1+1\neq 2) \vee (1+1\neq 3)$ (both (T))

③ $\neg((1+1=2) \vee (1+1=3))$

equiv to $(1+1\neq 2) \wedge (1+1\neq 3)$ (both (F))

④ $(\forall x\in\mathbb{R})\neg\left[(x<0) \wedge (\exists y\in\mathbb{R})(y^2=x)\right]$

equiv to $(\forall x\in\mathbb{R})\left[\neg(x<0) \vee \neg(\exists y\in\mathbb{R})(y^2=x)\right]$

equiv to $(\forall x\in\mathbb{R})\left[x\geq 0 \vee (\forall y\in\mathbb{R})(y^2\neq x)\right]$

(all (T))

Equivalences for $\Rightarrow$ and $\Leftrightarrow$

Prop'n For any statements P, Q the following equivalences hold:

① $(P\Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$

② $(P\Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$

③ $(P\Leftrightarrow Q) \Leftrightarrow (P\Rightarrow Q \wedge Q\Rightarrow P)$

Pf of ① + ②

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg P \vee Q$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T	T
T	F	F	F	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

$(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$	$(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$
T	T
T	T
T	T
T	T

Pf of ③: you try

↳ these equivalences can be useful for proving statements of the form $P \Rightarrow Q$ and $P \Leftrightarrow Q$.

Negating over $\Rightarrow$ and $\Leftrightarrow$

Prop'n the following logical equivalency hold:

① $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

② $\neg(P \Leftrightarrow Q) \Leftrightarrow [(P \wedge \neg Q) \vee (Q \wedge \neg P)]$

Pf: you try

↳ with these and our previous equivalency, we can now put any negated statement in "positive form"

<u>Def'n</u>: A statement P is in <u>positive form</u> iff any negation symbols in P only occur next to substatements that contain no connectives or quantifiers (i.e. negation symbols are "as inside as possible")

↳ our rules above allow us to find, for any P, a logically equivalent P′ in positive form.

<u>Ex's</u>: Let $E = \{2, 4, 6, \dots\}$
$O = \{1, 3, 5, \dots\}$

① $(5 \in O) \Rightarrow (6 \in E)$

equiv to: $\neg(5 \in O) \vee (6 \in E)$

which we can write: $(5 \notin O) \vee (6 \in E)$

all (T)