

(both are (T) :

Q has no multiplicative inverse)

→ "there is a red w/o a multiplicative inverse"

(54)

Negating Connected Statements

Prop'n For any statements P, Q the following logical equivalences hold (i.e. these statements are always true)

$$(1) \neg\neg P \Leftrightarrow P$$

$$(2) \neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$$

$$(3) \neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$$

De Morgan's Laws

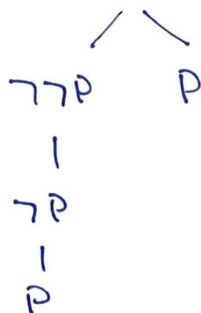
Pf: to prove, we use truth tables

(1)

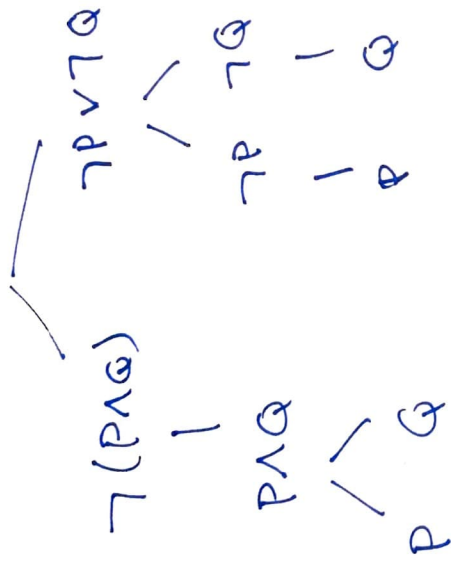
P	$\neg P$	$\neg\neg P$	$\neg\neg P \Leftrightarrow P$
T	F	T	T
F	T	F	T

always true

$$\neg\neg P \Leftrightarrow P$$



$$\textcircled{2} \quad \neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$$



Proof of $\textcircled{3}$

is similar

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$	$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

always true $\Rightarrow$

$\textcircled{3}$

Ex's ① $\neg\neg (1+1=2)$

(56)

equiv to $1+1=2$ (both (T))

② $\neg (1+1=2 \wedge (1+1=3))$

equiv to $(1+1 \neq 2) \vee (1+1 \neq 3)$ (both (T))

③ $\neg ((1+1=2) \vee (1+1=3))$

equiv to $(1+1 \neq 2) \wedge (1+1 \neq 3)$ (both (F))

④ $(\forall x \in \mathbb{R}) \neg [(x < 0) \wedge (\exists y \in \mathbb{R})(y^2 = x)]$

equiv to $(\forall x \in \mathbb{R}) [\neg(x < 0) \vee \neg(\exists y \in \mathbb{R})(y^2 = x)]$

equiv to $(\forall x \in \mathbb{R}) [x \geq 0 \vee (\forall y \in \mathbb{R})(y^2 \neq x)]$
(all (T))

Equivalences for $\Rightarrow$ and $\Leftrightarrow$

Prop'n For any statements P, Q
the following equivalences hold:

$$① (P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$$

$$② (P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$$

$$③ (P \Leftrightarrow Q) \Leftrightarrow (P \Rightarrow Q \wedge Q \Rightarrow P)$$

Pf of ①+②

(57)

P	Q	$P \Rightarrow Q$	$\neg P$	$\neg Q$	$\neg P \vee Q$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T	T
T	F	F	F	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

$(P \Rightarrow Q) \Leftrightarrow (\neg P \vee Q)$	$(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$
T	T
T	T
T	T
T	T

Pf of ③: you try

→ these equivalencies can be useful for proving statements of the form $P \Rightarrow Q$ and $P \Leftrightarrow Q$.

Negating over $\Rightarrow$ and $\Leftrightarrow$

Prop'n the following logical equivalency

hold: ① $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

② $\neg(P \Leftrightarrow Q) \Leftrightarrow [(P \wedge \neg Q) \vee (Q \wedge \neg P)]$

Pf: you try

↳ with these and our previous equivalencies, we can now put any negated statement in "positive form"

Def'n: A statement P is in positive form iff any negation symbols in P only occur next to substatements that contain no connectives or quantifiers (i.e. negation symbols are "as inside as possible!")

↳ our rules above allow us to find, for any P , a logically equivalent P' in positive form.

Ex's: let $E = \{2, 4, 6, \dots\}$
 $O = \{1, 3, 5, \dots\}$

$$\textcircled{1} (5 \in O) \Rightarrow (6 \in E)$$

equiv to: $\neg(5 \in O) \vee (6 \in E)$

which we can write: $(5 \notin O) \vee (6 \in E)$
 all (τ)