2 - <u>finite</u> sets can be defined by just writing all of their elements in brackets

- called <u>roster notation</u>

- e.g. if $A = \{2, 4, 6, \pi\}$

$B = \{\heartsuit, *, \pi\}$

Then $\pi \in A$ and $\pi \in B$

while $\heartsuit \in B$ but $\heartsuit \notin A$

↳ sets are determined by their elements; order and repetition don't matter.

e.g. if $A = \{1, 2, 3\}$

then $A = \{2, 1, 3\}$ and $A = \{1, 1, 2, 3\}$ too.

3 - sets can be elements of sets!

- e.g. if $A = \{1, 2\}$ $B = \{3, 4\}$

then $C = \{A, B\} = \{\{1, 2\}, \{3, 4\}\}$ is a legit set.

- <u>different</u> from $D = \{1, 2, 3, 4\}$

(C has 2 el'ts, D has 4)

Some Fundamental Sets

$\mathbb{N} = \{1, 2, 3, \ldots\}$ "natural numbers"
(for us: doesn't include 0)

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ "integers"

$\mathbb{Q} = \{\frac{m}{n} \mid m, n \text{ are in } \mathbb{Z} \text{ and } n \neq 0\}$ "rational numbers"

$\mathbb{R}$ = set of real numbers

$\mathbb{C}$ = set of complex numbers

$= \{a + bi \mid a, b \in \mathbb{R}\}$

So e.g. we have:

$0 \in \mathbb{Z}$ but $0 \notin \mathbb{N}$

$\frac{22}{7} \in \mathbb{Q}$ but $\frac{22}{7} \notin \mathbb{Z}$

$\pi \in \mathbb{R}$ but $\pi \notin \mathbb{Q}$

$i \in \mathbb{C}$ but $i \notin \mathbb{R}$

$\sqrt{-1}$

- the empty set is the unique set with no elements
- denoted $\emptyset$ or $\{\}$

not same as $\{\emptyset\}$ since this contains an el't

New sets from old ones

Set-builder notation: given a set X and a well-defined property P, can form the set Y consisting of all $x \in X$ with property P:

we write: $Y = \{x \in X \mid x \text{ has } P\}$

or $Y = \{x \in X \mid P(x)\}$

always need to specify the X from which the x's are drawn from

↳ called "set-builder notation"

Ex's ① can define $E = \{2, 4, 6, \dots\}$ by:

$E = \{n \in \mathbb{N} \mid n \text{ is a multiple of } 2\}$

or, more symbolically,

"such that"

$E = \{n \in \mathbb{N} \mid \text{there is } k \in \mathbb{N} \text{ s.t. } n = 2k\}$

② once E is defined, can use it to define other sets

e.g. define $O = \{n \in \mathbb{N} \mid \text{there is } k \in E \text{ s.t. } n = k - 1\}$

$= \{1, 3, 5, 7, \dots\}$

(3) the set over which you range is important:

$$\{x \in \mathbb{R} \mid x^2 - 2 = 0\} = \{\sqrt{2}, -\sqrt{2}\}$$

whereas:

$$\{x \in \mathbb{Z} \mid x^2 - 2 = 0\} = \emptyset$$

since no integers satisfy $x^2 - 2 = 0$.

Some more notation:

- for a given $n \in \mathbb{N}$, $[n]$ denotes the set $\{1, 2, \dots, n\}$
- e.g. $[5] = \{1, 2, 3, 4, 5\}$.

$\boxed{\text{Subsets}}$

- a set Y is a subset of X if for every element $y \in Y$ we also have $y \in X$.
- we write $Y \subseteq X$.
- Y is a proper (or strict) subset of X if $Y \subseteq X$ but $Y \neq X$.
- we (sometimes) write $Y \subsetneq X$ or $Y \subset X$ to indicate "Y is a proper subset of X"

– whereas $Y \not\subseteq X$ means "Y is not a subset of X"

<u>Ex's</u> ① $\{1,3\} \subseteq \{1,2,3,4\}$

<u>Why</u>: $1 \in \{1,2,3,4\}$ and $3 \in \{1,2,3,4\}$

It is a proper subset so we could write:

$\{1,3\} \subsetneq \{1,2,3,4\}$ or $\{1,3\} \subset \{1,2,3,4\}$

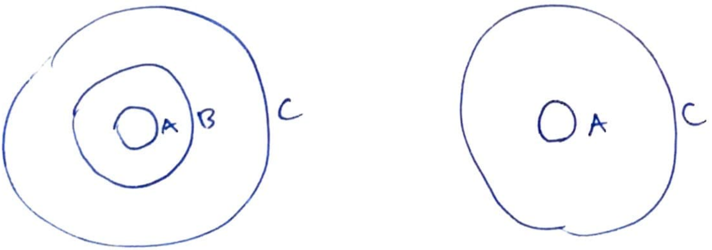
② $\{-10,3\} \not\subseteq \{1,2,3,4\}$

<u>Why</u>: $-10 \in \{-10\}$ but $-10 \notin \{1,2,3,4\}$

③ $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

<u>Notice</u>: "$\subseteq$" is a <u>transitive</u> relation

i.e. if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.



Let's <u>prove</u> this from the def'n of $\subseteq$