

Ma/CS 6c
Assignment #1
Due Tuesday, April 8 at 1:00 pm.

- (40%) **1.** (a) Prove the correctness of the following algorithm for recognizing when a given string S is a P -wff:

If $S = s_1 s_2 \dots s_n$, compute $w(s_n), w(s_n) + w(s_{n-1}), \dots, w(s_n) + w(s_{n-1}) + \dots + w(s_1) = w(S)$. If all these sums are ≥ 1 and $w(S) = 1$, then S is a P -wff; otherwise, it is not. (Recall that $w(p) = 1$, $w(\neg) = 0$, $w(*) = -1$, if $*$ = $\wedge, \vee, \Rightarrow, \Leftrightarrow$.)

Apply this algorithm to the strings:

- (i) $qp \Rightarrow \neg ttr \Rightarrow \wedge \neg stuv$;
- (ii) $\Rightarrow \Leftrightarrow \neg \wedge pq \vee \neg p \neg qs$.

If any of these strings is a P -wff, write down the corresponding wff.

- (b) Formulate an analogous algorithm for recognizing RP -wff.

- (40%) **2.** Show that unique readability holds for the formal language described below (whose grammatically correct strings we call R -wff):

- (i) Symbols: $p_0, p_1, \dots, \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$;
- (ii) Rules: (a) Each p_i is a R -wff;
 - (b) If A is a R -wff, so is $\neg A$.
 - (c) If A, B are R -wff, so are $A \wedge B, A \vee B, A \Rightarrow B, A \Leftrightarrow B$.

- (10%) **3.** Define the following function

$$r : \text{wff} \rightarrow \{0, 1, 2, \dots\}$$

by recursion:

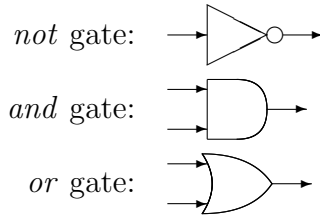
$$r(p) = 0$$

$$r(\neg A) = r(A) + 1$$

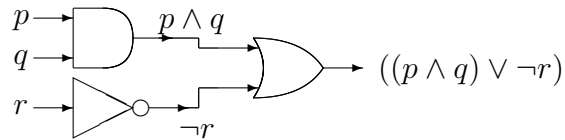
$$r((A * B)) = \max\{r(A), r(B)\} + 1.$$

Compute $r(A)$ for a couple of examples of your choice. What does $r(A)$ mean in terms of the parse tree T_A of the wff A ?

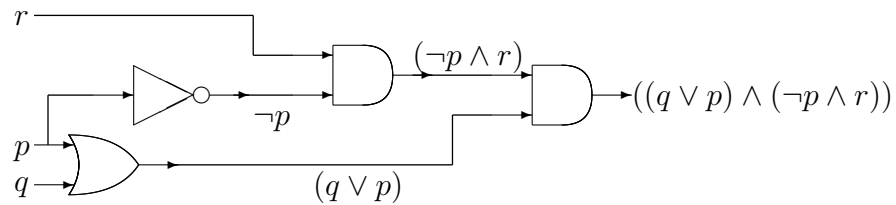
- (10%) **4.** Consider the following kinds of gates:



Every wff using only $\wedge, \vee, \neg$ corresponds to a circuit built out of these gates. For example, the wff $((p \wedge q) \vee \neg r)$ corresponds to the circuit:



and $((q \vee p) \wedge (\neg p \wedge r))$ corresponds to the circuit:



(a) Construct a circuit corresponding to the wff:

$$((p \wedge q) \vee (p \wedge (\neg q \vee r))).$$

(b) Find a wff corresponding to the circuit:

