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Abstract: We introduce a notion related to strong indecomposability for linear orders.

Say that a linear order X *absorbs* a linear order A if X has a partition $X = Y \cup A'$ such that $X \leqslant Y$ and $A' \cong A$. Here, $\leqslant$ means "embeds in." Equivalently, X absorbs A if there is a partition $X = X' \cup R \cup A'$ such that $X \cong X'$ and $A \cong A'$. We write $A \to X$ if X absorbs A.

The *spectrum* $\mathscr{S}_X$ of X is $\{A : A \leqslant X\}$, the class of orders that embed in X. The *absorption spectrum* $\mathscr{A}_X$ of X is $\{A : A \to X\}$, the class of orders absorbed by X. Since $A \to X$ implies $A \leqslant X$, we have $\mathscr{A}_X \subseteq \mathscr{S}_X$.

Say that X is *strongly splitting* if $X \to X$, or equivalently, if $\mathscr{A}_X = \mathscr{S}_X$. Say that X is *strongly indecomposable* if whenever $X = A \cup B$ is a partition of X, then either $X \leqslant A$ or $X \leqslant B$. By Leaf #8, if X is strongly indecomposable then X is strongly splitting.

Observation: $\mathscr{A}_X$ is an "ideal" of orders in the weak sense that:

i. If $B \subseteq A \in \mathscr{A}_X$ then $B \in \mathscr{A}_X$;
ii. If $A, B \in \mathscr{A}_X$, then there exists $C \in \mathscr{A}_X$ and a partition $C = A' \cup B'$ such that $A' \cong A$ and $B' \cong B$.

Question: Is there a linear order X such that $\mathscr{A}_X = \{A : A < X\}$?

Here, $A < X$ means $A \leqslant X$ and $X \not\leqslant A$.